A Duration-of-Stay Storage Policy in the Physical Internet

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1. INTRODUCTION

a. Introduction to Current Warehousing

In most situations it takes longer for a manufacturer to make and ship a product than the time a customer allots. In response to this, manufacturers produce more than the ordered amount and store these products in a warehouse to be shipped on an as-needed basis. This strategy consolidates the product, reduces response time to the customer, and aims to reduce the transportation costs associated with the shipping of products. Although the storage of products adds no physical value to the product, warehouses exist to reduce the overall cost of the system. Overall costs are reduced because companies no longer have to make everything according to a customer order; instead they mass produce and store products in warehouses. The optimization of the shipping and storage of these products is a major topic of research because, although warehouses currently do reduce transportation and overall system costs; these activities always have room for improvement and innovation.

Because of variability in operations and products today, there are different types of warehouses to accommodate these differences. In unit-load warehouses, items are stowed and picked in pallet quantities. In a break-bulk operation, items are usually received in bulk and broken down into smaller quantities. Upon customers’ orders, these smaller quantities are repackaged and shipped in various quantities. The identity of a break-bulk facility lies in that products are only broken down one level of storage: from pallet to case size. Last, in an order fulfillment warehouse, like the break-bulk facility, the pallets are broken down from pallet size. Unlike the break-bulk facility, they are broken down two levels from pallet to case size then from case to item size to be stored. As the workers travel through the picking area, they build orders by picking items or cases from the stored pallets. Manufacturers, importers, exporters, and more use all of these different types of warehouses.

For the majority of warehouses there is a distinct process that takes place as products arrive and are shipped. Upon unloading from trailers, pallets might be broken into case quantities or immediately put away in the storage area of the warehouse. Products at this point could be stored in pallet or case size. After some time in storage, the product would be picked, packaged, and shipped according the customer’s order specifications. These orders could be fulfilled in pallet, case, or individual piece quantities. If a pallet or case is moved from a reserve storage area
to a picking area in the warehouse, this activity is called a replenishment. Forklifts, pickers with carts, or even cranes moving above the picking area fulfill these activities (put-aways, picks, and replenishments) in the warehouse. It is important to make this step of the shipping process as efficient as possible.

There are also different types of storage or layout methods for the picking area in a warehouse. The most common type of storage is a shared random storage. In a shared storage policy, the number of storage locations in the warehouse is equal to the max of the sum of all incoming SKUs in one day over the planning period. Because SKUs share all the different storage locations, each SKU could randomly be stored in any location within the warehouse. The main question to ask when implementing a random storage policy is how many storage locations should be employed for the entire SKU set.

Fully dedicated storage is another storage policy in which every SKU gets a number of storage locations exclusively allocated to it. The space needed for the storage area is equal to the sum of the maximum daily values of each SKU in the system. A dedicated storage policy often requires more space, but also often requires less travel because the operator can strategically allocate more popular items to closer locations. So, given that a dedicated storage policy generally takes more space, the tradeoff is the increase in space versus the increase in efficiency.

Class-based storage is another popular type of storage in current warehouse operations. In this storage policy, classes are defined and those of a class are stored randomly within locations dedicated to the class. Products with a higher activity are stored closer to the loading and unloading docks within the warehouse. Although it requires more space than random storage and less than dedicated storage, it ensures, more often than not, shorter travel times because the picker will be travelling to these closer products more often. In turn, products with lower activity are stored farthest away from the loading and unloading docks in the warehouse. This could be implemented within a pallet rack where the higher traffic items would be located in a smaller but closer area to the loading and unloading docks. When considering the implementation of a class-based storage policy, there are two main questions to consider: how SKUs should be organized into classes, and how many storage locations should be dedicated to each SKU class.

Even though random storage takes up the least space, for a specified area it tends to have higher labor costs. Dedicated storage has the lowest labor costs, but tends to take up more space. This larger space requirement could potentially negate the labor cost improvement. Because the
class-based storage policy is a hybrid of the two previous storage policies, it aims to take advantage of the best aspects of each storage policy. It uses the advantages of the dedicated storage policy to help lower the labor costs, but uses the advantage of the shared storage policy to create a smaller storage area.

Duration-of-Stay Storage policy or DOS is the storage policy we will consider in our later models. DOS aims for the lowest possible storage requirements and the lowest possible labor costs. DOS is based on the information of how long the product or pallet will be stored in the warehouse as opposed to its SKU identifier. Upon arrival, the container is identified by its DOS (how long the product will be stored). An example is if the $DOS = 3$ days for a product and it arrives on Monday then it would need to be shipped on Thursday. Products with the shortest DOS would be located closest to the input/output point (I/O). As one moves farther away from the order shipping area, the DOS of products would also increase. We will only consider unit-load operations with a DOS storage policy.

b. Introduction to the Physical Internet

The Physical Internet (PI) is an open global logistics system founded on physical, digital, and operational interconnectivity through encapsulation, interfaces, and protocols of physical goods (Montreuil, 2011). It aims to improve by an order of magnitude the economic, environmental and social efficiency and sustainability of the way physical objects are moved, stored, realized, supplied, and used across the world. The PI shifts from private supply networks to an open global supply web with the intent of enlisting numerous different enterprises (Montreuil, 2011). This opens up the possibility for distributors, manufacturers, etc. to deploy their own products to a multitude of geographically disposed warehouses for the quick and efficient distribution of products. Most warehouses are currently utilized by at most ten different enterprises, with most of those warehouses only being used by one. With the implementation of the PI, warehouses would be able to serve many different enterprises. Just as the Digital Internet accomplishes this with information and data, the PI would allow increases in global utilization and communication in shipping, supply productivity, responsiveness, and adaptability.

Interconnectivity refers to the quality of a system to have its components seamlessly connected within the network (Montreuil, 2011). This interconnectivity ensures easy movement
of physical objects from one network to another, as well as their storage within all of its constituents just like the Digital Internet. The key to the universal interconnectivity is a high degree of collaboration amongst high-performance logistics centers and systems by exploiting world standard protocols to make the shipment and exchanging of physical objects quicker, easier, and cheaper through different transportation modes and routes (Montreuil, 2011). The PI generalizes and standardizes unloading, orientation, storage, and loading operations to help alleviate some of today’s biggest bottlenecks. Because of this universal interconnectivity, the PI has to work just as well in Chicago or Beijing as it does Los Angeles or Rome.

The PI also gears itself towards a unified, multi-scale conceptual framework that considers the big picture of the entire PI as one network all the way down to each individual process within one enterprise within the PI (Montreuil, 2011). This framework begins with an intra-center inter-process network to an intra-facility inter-center network all the way up to a worldwide inter-continental network. For example, with the intra-city inter-facility network, the PI allows for the ability to structure and empower efficient city logistics networks, helping products enter, move through, and exit cities while also helping to minimize the damage to those cities. The damages stemming from the freight logistics include pollution, noise, traffic, and safety issues. This multi-scale framework ensures the ease of safe shipment of products between cities, countries, and even different continents.

Also, the implementation of PI moves toward a distributed multi-segment intermodal transport of the physical objects within the PI (Montreuil, 2011). With the Digital Internet, packets of information do not travel straight from point A to point B. These packets of information travel from their origin to destination in the most efficient way possible according to the different routing algorithms and the varying blockage of networks. Unlike the Digital Internet, current distribution is dominated by either a point-to-point or hub-and-spoke method of transportation or shipment.

For example, say there is a trailer fully loaded with containers departing from Quebec with its destination being Los Angeles. With the current mode of transportation, one driver and truck might accomplish this job over a 120+ hour period. On the other hand, in the PI, loads would be handed off transfer points every 2-6 hours. Therefore, each of the needed 15 drivers would transport the containers in an already assigned PI-hub. The next driver and truck would then pick the containers up and move them another segment forward along in their journey to
Los Angeles. This process would be repeated until the containers would make it all the way to their destination in Los Angeles. Given the current way of transportation, a single driver would travel over 5,000 km to Los Angeles with at least 120 hours on the road one way. Instead, through the use of the PI’s intricate supply web, this same job would be completed by around 15 drivers with a total travel time of roughly 60 hours, or about half of the time.

One of the most important parts of implementing the PI is the encapsulation of goods into world-standard green, modular containers that would be called PI-containers (Montreuil, 2011). The PI would deal with these varying structural grade PI-containers instead of the physical object that is encapsulated by the PI-container, changing the way warehouses store these containers. These PI-containers would be stacked and interlocked together in the same way they are transported within the network rather than in racks. Second, it would change the operational components of the warehouse. Operations using forklifts and picking carts would become more standardized in these PI warehouses, thus again making these operations more efficient.

These PI-containers would range in size from cargo containers to tiny sizes with various conditioning capabilities like maintaining a specific temperature. They would be easy to dismantle, compose, and decompose to ensure the most efficient process of handling. A strong attribute of the PI-containers is their ability to be smart-tag enabled with sensors to allow for their proper identification, routing, and maintaining. Because of this, shippers and customers would be able to track their order or products throughout the shipping process.

The containers would be able to physically snap together to allow the maximum volumetric and functional density to increase integrity and durability. The encapsulated physical objects would be stored in the smallest possible PI-containers to avoid the storing and moving of air. Ultimately the PI-containers are foundational elements in enabling the implementation and success of the PI.

**c. Warehousing in the PI vs. Current Warehousing**

Because of the shift towards the encapsulation of the physical objects in the PI-containers, the global supply web, and high levels of protocols and standardization, the processes and decision-making associated with the layout and operations within the PI warehouses will be different than the current warehouse operations.
Although there is some standardization in today’s warehouses, from one warehouse to the next, even within the same country, there can be size differences in pallets or cases or even in the racks in which they are stored. In turn, because of these differences, the forklifts and carts associated with them also vary from one operation to the next. With the PI, standardization and the exploitation of different protocols and equipment lead to the goal of increased efficiency. All warehouses would be capable to receive, store, and ship out PI-containers to enable efficiency and use of the open global supply web (Montreuil, 2011). In current warehouse operations, products arrive, are broken down, and repackaged before being shipped. On the other hand, the PI-containers would be shipped from warehouse to warehouse with little to no modification or repackaging.

Upon arrival, containers will be identified and labeled differently in the PI than they are now in current operations. Currently, using SKU-based information, a company would design the layout of their warehouse (Gu, 2007). In contrast, in the PI, the SKU identifier would be irrelevant because operators handle the modular containers, rather than the actual product. Instead, the PI-containers would arrive, be labeled and identified with an expected ship-out date, which will determine its DOS. The PI warehouse layout would be designed based on each of the PI-container’s DOS. Using this information as will be addressed later; a variation of a DOS storage policy would then be able to be implemented.

Unlike many current warehouse operations, the PI-containers would be smart-tag enabled, giving them tracking and monitoring capabilities. Even though in today’s warehouse operations these smart-tags might track a certain SKU, in a PI warehouse these smart-tags would track where a certain PI-container is located within the network (Montreuil, 2011). This proves to be helpful because with the DOS storage policy, the PI-containers do not necessarily have a specific location to be stored in the warehouse like the SKUs do. Because of the smart-tags, the warehouse operator would be able to accurately track PI-containers throughout the system as well as vacant and occupied locations in the warehouse.
2. LITERATURE REVIEW

When looking to justify the need for a storage policy for the PI, it was vital to take a look at past storage policy research. Because of the nature of the PI, we only consider storage policies relating to unit-load operations. The first storage policy examined was the dedicated storage policy introduced as the Cube-per-Order (CPO) index in (Heskett, 1963), which is aimed at minimizing costs in the staging area of a warehouse. It establishes a quantitative tradeoff between the dual objectives of placing closest to the order shipping area those items taking up the least space, and also those items that are the most popular. The CPO index is the ratio of the average number of cubic feet of storage required per order multiplied by the average number of orders to be received per order multiplied by the average number of orders to be received per shipping day during the time horizon to the average number of orders per shipping day (Kallina, 1976).

As mentioned before, the key to any fully dedicated storage policy is to provide space enough for the sum of the maximum amount for all the SKUs. The CPO index is a “turnover-based policy” in which the storage is directly dependent on how long it takes for the inventory to completely turn over. Also, this type of policy looks to exploit the higher-activity products by placing them closer to the input/output (I/O) point.

When comparing fully dedicated storage with random storage, there are some key tradeoffs and differences. For equally sized warehouses, a fully dedicated policy, on average, results in shorter traveling distances as opposed to a random storage policy. When all the items have the same probability mass function for the selection of a dock and the same number of storage locations needed, dedicated storage is optimal (Geotschalckx, 1990). As the ABC curve for the dedicated storage policy becomes more and more skewed, the gap between dedicated and random storage widens with respect to traveling distances (Caron, 2000).

There is one instance in which both dedicated and shared storage policies yield the same result in terms of size of the storage area in the warehouse: when the full throughput of the system arrives all on the same day. The summed maximum daily throughput for each SKU would be equal to the maximum total throughput over the planning period also, so the storage area for both storage policies would be the same size, yielding the same area requirements.
When looking at different storage policies, we lastly consider the DOS storage policy (Goetschalckx, 1990), which we use as our storage policy. The DOS storage policy exploits the shared storage idea that different products might share a location at different points in time. Goetschalckx explains, with DOS storage policy, it is more efficient to store the items with the smallest DOS closest to the input/output (I/O) point. As one travels farther into the picking area farther away from the I/O point, the DOS of the nearby items increases.

A DOS shared storage system is perfectly balanced if, for any period, the number of departing units that have a DOS of value, $p$, is equal to the number of arriving units that have a duration of $p$, for all $p$. For this perfectly balanced system, a shared storage policy minimizes both travel time and required storage space. A more realistic adaptive shared storage policy might have classes based on average arrivals and departures of items with different durations of stay (Goetschalckx, 1990). For $n_p$ equal to the number of units with DOS equal to $p$, we define a zone size of $n_p \times p$ for DOS $p$. Using this method, the storage area could be too small or too big from day-to-day, but will, on average, create a balanced storage area.

The DOS storage policy, at its best, produces a relative time travel savings of about 15% when compared to a shared random storage policy. When compared to a dedicated storage, the DOS storage policy also produces an improvement of about 37.5% in required rack size (Goetschalckx, 1990). Because the DOS storage policy takes advantage of a shared storage policy it requires the same size warehouse as the other shared storage policies. These reductions can and will vary with different sets of parameters and assumptions. In this case, Goetschalckx considered three independent factors: reorder quantity, the number of products stored in the rack, and the average demand interarrival time. If it can yield this great of a reduction in travel, it is worth reviewing when considering how to store the PI-containers.

The degree to which the DOS storage policy is more efficient is directly dependent on the “balance” of the system. This “balance” refers to the number of incoming and outgoing SKUs each day in the warehouse. A perfectly balanced system would have the same number of incoming and outgoing SKUs so that every storage location is optimally utilized; every storage location would be full at the end of the day. As the gap between daily S/R numbers is minimized, the system becomes increasingly balanced.

The results given by Goetschalckx are promising for the implementation of DOS storage policy because it is able to reduce the size of the picking area as well as the labor costs.
associated with the travel distance. Although promising, it requires the most information when compared to the other storage policies; namely, the DOS of incoming containers is required (the majority of operations today are not supplied this information). Our policy looks to capitalize on the interconnectivity and high-level of communication within the PI to solve this problem, so the PI will be able to fully take advantage of the benefits of the DOS storage policy.

Goetschalckx explains, “The best way to implement such a policy is not clear at this point.” Our contribution begins here, by defining a DOS storage policy for the PI. In addition to defining the storage policy itself, we will look into the best way to implement this storage system in the PI. In the following sections we will discuss how, using the information that the PI provides us, we can implement this storage system.

3. PROBLEM STATEMENT

a. Overall Problem

As of now, the most appropriate storage policy for warehouses in the PI is unclear. When searching to define a good storage policy for warehouses in the PI, the minimization of the cost of the size of the picking area and the labor costs is the focus. This storage policy would be directly based on information we already know or receive upon the arrival of unit-containers. For example, the container’s DOS would be easily accessed from the smart-tags on the unit-containers. The smart-tags would also hold vital information such as where the container is coming from and where it is going (Montreuil, 2011).

For a unit-load operation with one central I/O point, our storage policy will be a form of the Duration of Storage policy (Goetschalckx, 1990). Containers leaving sooner would be stored in “good” locations in the storage area and containers leaving later would be stored in “bad” locations. The location for which a container should be stored will be called the target location. For our storage policy, defining which locations are “good” and “bad” in the warehouse proves to be fairly trivial. “Good” locations would be closer to the (I/O) point and the “bad” locations would be farther way from the I/O point. The difficulty comes in the decision of which containers should be stored in the “good” locations and which containers should be stored in the “bad” locations.
When considering this storage policy, we need to address the tactical analysis of the problem by asking ourselves, “How much space do we allocate to each DOS target value?” By addressing this, we help to create a more balanced warehouse by making sure each DOS value has the adequate number of locations allocated to it. The next question addresses the operational aspect of our storage policy: “Where should this particular container be stored?” We want to store the container, based on its DOS, in the most efficient and cost-effective location with respect to the size of the picking area and labor costs. Ideally, each container would be stored in one of the storage locations allocated to its unique DOS, but because of the variance of arrival policy that will not always be possible. Thus, the rules that define our DOS policy will affect its performance.

b. Parameters and Response Values

For each planned warehouse within the PI, there is a set of measurable factors or features that characterizes the system, called parameters. On the other hand, response values are used to evaluate the validity and performance of the model for the system. In addition to these benefits, the response values characterize the output of the system, so the parameters are inputs to the system and the response values are outputs of the system. The response values aid in the prediction of not only the system we have created, for future systems in the PI, and ultimately the comparison of different systems with unique parameters. We are able to interpret and evaluate then how a different set of parameters might change the model with respect to the response values. Through the varying of parameters and the interpretation of the response values, we aim to help in the answering of a few important questions.

c. Important Questions to Answer

Answering these questions would aid in the decision-making process for warehouses within the PI. The thoughts and answers to these questions give some insight in how to appropriately plan and operate a PI warehouse with a set of unique parameters based on the performance of the response values.
1. **How do the response values change as the warehouse size increases?**

   Considering two different warehouse quantities and assuming the throughput-to-warehouse-size ratio is held constant, how much do the response values change in response to an increase in warehouse size? E.g., If the warehouse grows by 10%, how do the response values improve or degrade? This type of information would prove to be important during the planning and operating stages of the warehouse.

2. **As the other parameters change, do the response values change proportionally?**

   Excluding the size of the warehouse, do the response values respond proportionally as each of the parameters change? It is also important to consider the direction of this change. As a parameter changes, do the response values change inversely or in the same direction? Having knowledge of this change would help tremendously in determining how to take action for an anticipated significant parameter change.

3. **Do the response values degrade as the variance of a parameter’s distribution increases?**

   As the variance of a parameter’s distribution increases, do the response values degrade? We increase the variance of a parameter’s distribution by varying the minimum and maximum values of the parameter. We look to uphold our hypothesis that the response values will degrade as this variance increases. So, as a variance increase is anticipated, one will need to know how to plan accordingly, because there will be some change in response values associated with this increase in variance.

4. **Do the response values prove to be more sensitive for very high/low parameter values?**

   This question considers how response values change as two different parameters interact. For a very high or very low parameter value, are the response values more sensitive to a change in other parameters? Example: Consider a very high arrival rate of unit-containers to a warehouse on a day. As other parameters are changed, do response values react similarly as when the arrival rate was neither high nor low? Because of seasonality, there might be times when there might be a higher than average throughput. It is important for a warehouse manager to know how the response values would change in any and every case.
5. **Do designs exhibit any economics of scale?**

Put simply, do we see any advantage of having a much larger warehouse because of a higher volume of activity within the warehouse? Because of the increased area, we do realize there will be an increase in the average travel distances when moving from a small warehouse to a larger one. But because of the increased number of storage locations in each target zone, we conclude there stands a better chance of each container being correctly stored in its target zone in the large warehouse when compared to the smaller. When the container is correctly stored in its target zone, we believe our storage policy will help to improve the operational performance. In this case, the average distance would not grow as much as one might think when considering the increase in the size of the warehouse. This is important to consider in the case that one might consider building one large warehouse to handle all the throughput rather than having two smaller warehouses.

Even though these questions are not all encompassing, they are important in the planning and analysis of a warehouse system in the PI. Obtaining the answers to these questions will not only help in analysis, but also hopefully lead to some more even interesting and important questions about the system. Addressing these questions will provide some insight and foundation of how our DOS storage policy in the PI will react to changes in the range of the inputs and in the inputs themselves. Changes in most systems are likely, but changes in the PI are inevitable, as a PI warehouse is not built based on one company’s activities.

4. **METHODOLOGY**

**a. Model**

Given our formulated DOS storage policy, we set out to build a model for the average one-way put-away distance in a unit-load warehouse in the PI. A put-away operation is comprised of the travel distance parallel to the side of the warehouse as well as the travel distance parallel to the picking aisles. This travel would be directly dependent on the length of the aisles as well as the width of the picking area in the warehouse. We make the assumption that the height of our warehouse is fixed at one level (i.e., no associated vertical travel) to simplify our model.
The model will calculate an estimate of the one-way put-away travel distance for a given PI-container for our DOS storage policy with the goal being to minimize the travel.

Parameters:

\[ W_{uc} = \text{center-to-center unit-container width} \]
\[ W_A = \text{center-to-center aisle width} \]
\[ T = \text{one-way put-way travel distance} \]
\[ N_A = \text{location of aisle with respect to the center aisle} \]
\[ N_{SL} = \text{location of pick within the aisle from end closest to I/O} \]

![Diagram of warehouse layout with W_{uc} and W_A measurements.]

**Figure 1: Warehouse Layout with W_{uc} and W_A measurement**

Model:

\[ T = N_{SL} \times W_{uc} + N_A \times W_A \]
Example:

\[ W_{uc} = 4 \text{ feet} \]
\[ W_A = 20 \text{ feet} \]
\[ N_A = 3 \text{ (3 aisles away from center)} \]
\[ N_{SL} = 7 \text{ (7th storage location within the aisle)} \]

\[ T = 88 \text{ feet of one-way put-away travel distance for this PI-container} \]

b. Simulation

The ability to simulate a system using the storage policy proves important because of the ability to relax and vary some of the model’s parameters to see how these parameters change the system with respect to the different response values. Beginning with a simple system such as a two-sided aisle with no stacking will help to more clearly explain and understand a more realistic complex system.

Because the PI is not a reality yet, we will generate the needed parameter values. Our system would need to know the expected time between arriving PI-containers. With this expected time between arrivals, we will be able to calculate the number of expected incoming unit-containers arriving to the warehouse in a day, \( \lambda \). The size of these shipments with each unique DOS is another key piece of data that will be required. This data will help in knowing the distribution of containers that will need to be stored for each DOS during a certain period of time. Using these data, the system is simulated using the put-away model to calculate the above-mentioned one-way put-away travel distance.

Simulation Design

Building a simulation model was the next step in finding out what a warehouse in the PI might resemble. This simulation model shows a quick snapshot of what actually goes on each day in the warehouse. Our formulated model uses Excel VBA to create a grid of cells resembling a set of storage locations within a warehouse. The simulation captures each incoming unit-container, where it is stored, how long it is stored, and when it leaves the warehouse, along with the one-way put-away travel defined earlier.
First, choosing a set of base parameters in which to start building the simulation model was important to be able to perform adequate sensitivity analyses. These analyses would then be conducted by manipulating the set of parameters. For the smaller of the two warehouses to be considered, the parameters include, the aisle depth in number of storage locations, $D$, and the number of aisles, $A$, and the total number of storage locations, $SL$:

\[
D = 21 \text{ storage locations} \\
A = 9 \text{ two-sided aisles} \\
SL = 2 \cdot D \cdot A = 378 \text{ storage locations}
\]

Also, the physical dimensions of the warehouse based on the two needed measurements:

\[
W_{uc} = \text{center-to-center unit-container width} = 4 \text{ feet}, \\
W_{A} = \text{center-to-center aisle width} = 20 \text{ feet}.
\]

Given these two sets of parameters, it ensures that the width-to-depth ratio of the warehouse is approximately 2.0.

The first two parameters, average number of incoming unit-containers per day and the DOS of the unit-container, are both modeled by a discrete triangular distribution. The expected number of incoming unit-containers, $\lambda$, is determined according to the size or number of storage locations within the warehouse as described later. The following describes the three different values in the distribution of the DOS values:

\[
DOS = \text{duration-of-stay value for incoming unit-containers} \\
DOS_{\text{min}} = 1 \text{ day} \\
DOS_{\text{mode}} = 5 \text{ days} \\
DOS_{\text{max}} = 9 \text{ days}
\]

Figure 2 shows the breakdown of the percentage of each DOS:
These DOS values are not necessarily realistic; rather they are an estimate of how long unit-containers might stay in the warehouse in the PI. As more realistic information about the PI is discovered, the value and distribution of these DOS values should be revisited.

Because the initial DOS of each incoming unit-container in our model would be any integer value from 1 to 9, the most logical choice would be to designate nine different Target Zones (TZ) in the warehouse. While this seems simple, it can become quite complicated because of the high number of zones. So to simplify the process, we decided to designate five different target zones in the warehouse. The set of TZs are characterized as follows:

\[ TZ_A = \text{Target Zone for all unit containers with a DOS} = 1 \text{ or } 2; \]
\[ TZ_B = \text{Target Zone for all unit containers with a DOS} = 3 \text{ or } 4; \]
\[ TZ_C = \text{Target Zone for all unit containers with a DOS} = 5 \text{ or } 6; \]
\[ TZ_D = \text{Target Zone for all unit containers with a DOS} = 7 \text{ or } 8; \]
\[ TZ_E = \text{Target Zone for all unit containers with a DOS} = 9. \]
The size of each TZ is calculated according to the distribution of the number of unit-containers with a particular initial DOS. The model initially ran by creating a set of 378 storage locations with unique DOS values according the previously mentioned discrete triangular distribution. Calculating the average number of occurrences for each DOS as a percentage of the total number of occurrences, this percentage is used to size each TZ according to the summed percentage for the DOS in each TZ. For example, the DOS values corresponding to TZ_A make up about 7% of the total number of incoming unit-containers. Because 24 is approximately 7% of 378, TZ_A would get about 24 storage locations for incoming unit-containers. This sizing was completed in hopes that over time each incoming unit-container would be successfully stored in its TZ.

Incoming unit-containers are stored in the TZ according to its DOS. An example being, if a unit-container arrives with $DOS = 6$, it would be stored in TZ_C. The PI-containers with lower DOS values will be stored closer to the I/O point because these storage locations are visited more frequently and ultimately stay in the warehouse the shortest amount of time. Considering this, TZ_A is stationed closest to the I/O point and TZ_E is farthest from the I/O with TZ_B, TZ_C, and TZ_D respectively stationed between TZ_A and TZ_E.

At the beginning of each simulation step, the model generates a set of incoming unit-containers according to the previously mentioned discrete triangular distribution (with mode = $\lambda$). Each incoming unit-container is then assigned a DOS and placed in the closest open spot in its TZ, assuming there is an open location. All incoming unit-containers follow this procedure until they have been placed in the warehouse or until there are no open storage locations. As soon as one of these situations occurs, the model is finished running for that “day.”

At the end of each day or simulation step, the DOS is reduced for all stored unit-containers to represent the end of the day. When this happens, the “day” is over, and the unit-containers with an updated $DOS = 0$ are shipped. The model proceeds to the next step with a new set of incoming unit-containers, each with its own DOS value. Once this cycle of days starts, it continues for the predetermined run length, emptying (shipping out) and re-filling (storing) the storage locations in the warehouse with unit-containers and their DOS.

When the unit-containers are generated, there are six different cases that can happen. Figure 3 gives a graphical representation of the scenario when there are five TZs corresponding to DOS values from 1 to 5. The incoming unit-container has a $DOS = 3$. The following cases
generalize this decision-making process. TZ$_{i}$ corresponds to the TZ the incoming unit-container should be stored.

Case 1: The unit-container is stored TZ$_{i}$ because there is an open storage location in it.

Case 2: There are no open storage locations in TZ$_{i}$. The unit-container is then stored in TZ$_{i-1}$.

Case 3: In the case that TZ$_{i-1}$ either does not exist or has no open storage locations in it, the unit-container should then be stored in TZ$_{i+1}$.

Case 4: In the case that TZ$_{i+1}$ either does not exist or has no open storage locations in it, the unit-container should then be stored in TZ$_{i-2}$.

Case 5: This logic extends until all TZ$_{i-j}$ (TZs closer than TZ$_{i}$) and TZ$_{i+j}$ (TZs farther away than TZ$_{i}$) either don’t exist or are completely full. In this case, the unit-container is not stored in the warehouse and a travel penalty is incurred.
Figure 3: Graphical Representation of Decision-Making when Storing Unit-Container with $DOS = 3$ with Five Target Zones
In this last case, there are more incoming unit-containers than there are open locations in the warehouse; thus, some leftover unit-containers do not have a location to be stored. The simulation model incurs a “penalty” to account for the inconvenience of having leftover unit-containers that cannot be stored. This penalty is subjective; it could vary from one distribution center to the next depending on how needed space is accommodated. For our case, the penalty was set to 125% of the one-way travel to the farthest storage location within the warehouse. This penalty percentage (125%) is kept constant over the two warehouses to ensure consistency in the interpretation of the response values from the small warehouse to the large warehouse.

For the predetermined number of storage locations, we need to determine an appropriate value of $\lambda$ to create a balanced warehouse as discussed earlier. Because we define the different parameters for the distributions, the nature of the problem is deterministic and the answer is quite trivial. The following shows the calculations for the value of $\lambda$ that would create a balanced system, on average:

$$\lambda = \frac{SL}{\text{Average DOS}} = \frac{378}{5} = 75.6 \text{ unit containers}$$

While running the simulation over a 210-day period, we found this mean to be about 75.6 unit-containers as shown above. Figure 4 shows the full breakdown of the number of incoming unit-containers each day over the run period:
Figure 4. Number of Incoming Unit-Containers per Day Distribution  
(minimum = 21, mode = 76, maximum = 131)

For a completely balance warehouse, it would be completely full with no leftover unit-containers at the end of every day. The warehouse would be fully utilized and no travel penalty due to lack of space would be incurred. It is near impossible to have fully balanced warehouses even with setting λ as above because even though λ balances, on average, the day-to-day variation leads to an imbalance. We model a warehouse to understand the system and the DOS storage policy. The manipulation of these parameters for our system will help to be able to understand how the storage policy might change when the PI system changes. Not every PI warehouse is going to be exactly the same; warehouses will have different values for each of these assumptions that dictate the efficiency and feasibility of our storage policy.

Specific Parameters

There are three different specific parameters we consider for the interpretation and analysis of our model. These realistic parameters are inputs for the warehouses within the PI. Through these parameters we will look to answer the previously discussed “Important Questions to Answer.” The following defines the three different parameters:
\[ \lambda = \text{Average number of incoming unit-containers per day} \]
\[ V = \text{Variance of number of incoming unit-containers per day} \]
\[ SL = \text{Number of storage locations within the warehouse} \]

Adhering to a discrete triangular distribution, we assume the distribution curves modeling the number of incoming unit-containers per day and DOS values are symmetrical. The “steepness” of the distribution curve for the number of incoming unit-containers per day determines \( V \), so as the range of these incoming unit-containers increases, so does \( V \). Because the warehouse operator will be able to accept and reject business based on warehouse activity, this number of unit-containers is not totally unpredictable. Despite this, there will still be some variation in the number of incoming unit-containers from day-to-day. Given different values of \( \lambda \) and variation in the number of incoming unit-containers, the response values will react differently. Knowing \( V \) will aid in planning for not only the size of the warehouse but also the number of workers and equipment needed.

The last parameter to be discussed and considered is \( SL \). Because the number of storage locations determines the size or storing capacity of the warehouse, we will consider two different-sized warehouses. For our case, we will simplistically name these two warehouses small and large. Defining multiple warehouse sizes is important because there will be no “cookie-cutter” warehouse within the PI. There will need to be some small and some large warehouses. For planning and analysis purposes, one would need to know how a warehouse would perform depending on its size.

**Specific Response Values**

The specific response values are directly dependent on the above specific parameters and how they vary. We will look to address the “Important Questions to Answer” in relation to these specific response values:

\[ TD = \text{Average cumulative travel distance (including penalty) over the 210-day run length} \]
\[ P_{WH} = \text{Average percent of unit-containers that cannot be stored in the warehouse} \]
\[ P_{TZ} = \text{Average percent of unit-containers that cannot be stored in their TZ} \]
For statistical significance, all response values are calculated over 25 replications with a run length of 210 days with a warm-up period length equal to one run of 25 replications of 210 days. A significant amount of cost in a warehouse can be attributed to the labor and equipment needed to complete this one-way horizontal put-away travel. The higher the travel, the longer it takes to complete the work, and the greater the labor force needed to complete the travel. Through the use TZs, our storage policy aims to lower $TD$ because unit-containers with lower DOS values are stored closer to the I/O point. Through the manipulation of the parameters, we capture how $TD$ reacts in response to these changes.

Because of the earlier discussed penalty associated with having this surplus of incoming unit-containers, it is desired to minimize the $P_{WH}$. Because of this penalty, it is better for all of the incoming unit-containers to be stored in a storage location. It would be easy to completely eliminate this by significantly oversizing the warehouse in comparison to the $\lambda$ to ensure that every unit-container would make it into a storage location each day. In reality, this trade off becomes the minimization of the warehouse area versus the minimization of $P_{WH}$.

Even though TZs are sized with the goal that each unit-container would be stored in its TZ, we realistically know this is not always the case. In the majority of cases there will be some percentage of unit-containers that are not stored in their TZ; i.e., $P_{TZ} > 0$. Because of this goal, we look to minimize $P_{TZ}$ in the hopes that the majority of unit-containers are stored in their TZ, thus lowering $TD$.

Whenever a unit-container has to be stored in a closer TZ, it takes up a storage location a unit-container with a lower DOS should have. Even though it requires less travel for the current unit-container to be stored in the closer TZ, it will end up staying in the TZ longer than the rest of the unit-containers in the TZ. This takes up a spot longer and pushes out another unit-container that should be stored there. Whenever this occurs, the unit-container pushed out usually, in the short term, ends up being stored in a more distant TZ. This creates a higher initial travel distance, and also creates an imbalance in the TZs, which may persist thus, increasing the chances arriving unit-containers will not make it into its TZs.

On the other hand, a unit-container could be stored in a more distant TZ; whereas it would require more initial travel to put away the unit-container. Just like the previous case, storing the unit-container in a more distant TZ also creates an unwanted imbalance in the DOS of
the unit-containers for the TZs. Ultimately both of these cases end up creating unnecessary travel
within the warehouse. Because of this unnecessary travel, it is best to have the lowest possible
$P_{TZ}$ for a given size warehouse.

To answer our “Important Questions” we will need to know in which direction the
response values change and by what magnitude. Seeing how these response values change
according to changes in the parameters will go a long way in addressing the “Important
Questions” and help to give an idea of how a DOS storage policy in the PI might behave and
how to plan and operate the warehouse accordingly.

5. RESULTS

Addressing the results of the simulation model, we want to use the data to help answer our
questions about the system. As mentioned before, to thoroughly answer these questions is helpful
in understanding a DOS storage policy in the PI and determining a decision-making process for a
warehouse. These questions should be answered concisely through the interpretation and
presentation of the relevant data.

a. Validation of Model

We first present the data from our simulation runs in a series of graphs to validate our
basic understand of the system.

As intuition and Figure 5 suggests, with a higher $\lambda$ comes an increase in $TD$ over the run
period. Also observed is that as $V$ increases for each $\lambda$, the travel distance increases.
Figure 5. TD for the Smaller Warehouse with SL = 378

Next, (for the smaller warehouse) as Figure 7 suggests, as $\lambda$ increases so does $P_{WH}$. In the same respect, as $V$ increases, this $P_{WH}$ increases as well.

Figure 7. Average % Not in Warehouse
the Smaller Warehouse with SL = 378
Lastly, (for the smaller warehouse) Figure 6 shows that as $\lambda$ increases, $P_{TZ}$ decreases. The full analysis of this observation is addressed later. Also observed in this chart is the suggestion that as the variability in $\lambda$ increases $P_{TZ}$ decreases.

![Average % Not in Target Zone](image)

**Figure 6. Average % Not in Target Zone for the Smaller Warehouse with $SL = 378$**

b. Answers to Important Questions

1. **How does $P_{TZ}$ change as the warehouse size increases?**

   As the size of the warehouse increases with a constant ratio of storage locations to $\lambda$, consider how $P_{TZ}$ changes. In response to an increase in the warehouse size, $P_{TZ}$ decreases. As mentioned, the ratio of $SL$ to $\lambda$ is the same for the smaller and the larger warehouse. This is to ensure that the throughput is proportional for each warehouse.

   For a low $V$ and low $\lambda$, the smaller warehouse has about an average $P_{TZ} = 7\%$ but the larger warehouse has an average $P_{TZ}$ closer to $2\%$. Similarly, with a medium $V$ and medium $\lambda$, similar results are observed from the response value. Figure 8 illustrates this observation:
When addressing the change in $P_{TZ}$, we consider why this change is observed. First, the aforementioned ratio is held constant, so we know $\lambda$ for the larger warehouse will be a considerable amount larger than $\lambda$ for the smaller warehouse. Along with a larger $\lambda$, the number of spots within each TZ increases as well.

For the smaller warehouse, we consider how $P_{TZ}$ changes in response to an increase in $V$. It is observed as $V$ increases, $P_{TZ}$ decreases and $P_{WH}$ increases. Thanks to an increase in $P_{WH}$, on average, there’s a greater chance there will be more open storage locations at the end of each day. With more spots open comes a higher chance each unit-container will make it into the TZ. Because of this, $P_{TZ}$ decreases for the smaller warehouse. For the larger warehouse, this observation does not hold true; $P_{TZ}$ slightly increases as $V$ increases. Unlike with the smaller warehouse, the larger warehouse exhibits a decrease in $P_{WH}$ for an increasing $V$. As $V$ increases, slightly more and more unit-containers are able to be stored in the warehouse each day because of the increased chance of having more open storage locations.

As more unit-containers make it into spots in the warehouse, the warehouse becomes closer to completely full each day. As the warehouse gets closer to being full, there are less open
spots within each TZ. Of course, when there are less open spots in the TZ, there is less chance that an incoming unit-container will be stored in its TZ. Because of this, there is an increase in the percentage of unit-containers that will need to be stored in a different zone other than its TZ. So, as the warehouse gets larger, it becomes more efficient with respect to the percentage of unit-containers that make it into their TZ.

2. As \( \lambda \) changes, does TD change proportionally?

Basically, when there is a specific level of increase or decrease in \( \lambda \), does TD change proportionally? From observation of the simulation model results, the TD does not change proportionally. Our initial hypothesis would be if the \( \lambda \) increased by 10% then the TD might also increase by 10%; this proved to be untrue. It is assumed for more incoming unit-containers, then there will be an increase in TD. This increase in TD can be attributed to the fact that there are more put-away trips required for the extra incoming unit-containers that need to be put away.

The increase in TD is a much higher percentage than that of the increase in \( \lambda \). When moving from a \( \lambda = 65.6 \) to 75.6 (15% increase), there is about a 24% average increase in TD. For an increase from a \( \lambda = 75.6 \) to 85.6 (13% increase), there is about a 29% average increase in TD. Both of these changes occur with a low \( V = 4.2 \). Both of these increases in \( \lambda \) are similar, so it is noted the increase in TD for these two instances is also similar. The full set of data can be seen in the following Tables 1 and 2.

**Table 1: TD for \( \lambda = 65.6 \) and \( \lambda = 75.6 \), as well as the percent change in TD for a changing V in the smaller warehouse.**

<table>
<thead>
<tr>
<th>Smaller Warehouse</th>
<th>Ratio of Lowest V</th>
<th>( \lambda = 65.6, TD )</th>
<th>( \lambda = 75.6, TD ) (15.2% inc.)</th>
<th>% Change in TD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
<td>792,256</td>
<td>983,414</td>
<td>34%</td>
</tr>
<tr>
<td></td>
<td>9.0</td>
<td>903,464</td>
<td>1,196,707</td>
<td>32%</td>
</tr>
<tr>
<td></td>
<td>25.0</td>
<td>1,122,936</td>
<td>1,465,819</td>
<td>31%</td>
</tr>
<tr>
<td></td>
<td>49.0</td>
<td>1,399,466</td>
<td>1,754,876</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>80.9</td>
<td>1,697,563</td>
<td>2,075,272</td>
<td>22%</td>
</tr>
<tr>
<td></td>
<td>120.2</td>
<td>2,019,050</td>
<td>2,405,290</td>
<td>19%</td>
</tr>
<tr>
<td></td>
<td>168.8</td>
<td>2,349,439</td>
<td>2,743,241</td>
<td>17%</td>
</tr>
</tbody>
</table>
Table 2: $TD$ for $\lambda = 75.6$ and $\lambda = 85.6$ as well as the percent change in $TD$ for a changing $V$ for the smaller warehouse.

<table>
<thead>
<tr>
<th>Smaller Warehouse</th>
<th>$\lambda = 75.6$, $TD$</th>
<th>$\lambda = 85.6$, $TD$ (13.2% inc.)</th>
<th>% Change in $TD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of Lowest $V$</td>
<td>983,414</td>
<td>1,306,310</td>
<td>33%</td>
</tr>
<tr>
<td>1.0</td>
<td>1,196,707</td>
<td>1,543,629</td>
<td>29%</td>
</tr>
<tr>
<td>9.0</td>
<td>1,465,819</td>
<td>1,824,836</td>
<td>24%</td>
</tr>
<tr>
<td>25.0</td>
<td>1,754,876</td>
<td>2,135,530</td>
<td>22%</td>
</tr>
<tr>
<td>49.0</td>
<td>2,075,272</td>
<td>2,458,977</td>
<td>18%</td>
</tr>
<tr>
<td>80.9</td>
<td>2,405,290</td>
<td>2,796,767</td>
<td>16%</td>
</tr>
<tr>
<td>120.2</td>
<td>2,743,241</td>
<td>3,133,369</td>
<td>14%</td>
</tr>
</tbody>
</table>

Because of this observation, we can conclude that this consistent change in $TD$ is not necessarily dependent on the minimum and maximum $\lambda$; rather it is more dependent on the percentage increase or decrease in $\lambda$.

Now knowing the increase in $TD$ is not of the same scale as the increase in $\lambda$, consider how to further interpret this data. When running the model, for a constant percentage increase in $\lambda$, there is a change in the percentage increase in $TD$ only when $V$ is increased or decreased. This fact will then help to understand and address Question 4.

3. Do response values degrade as the variance of $\lambda$ increases?

As $V$ increases, it is important to interpret the reaction of the three different response values: $TD$, $P_{TZ}$, and $P_{WH}$. For the response value to degrade, its value undesirably increases or decreases; the definition of degrade could be different for each of response value. As $TD$ degrades, the value increases because more travel is undesirable. On the other hand, for both of the other response values, to degrade is for their values to increase because it is best to have all unit-containers successfully be stored in their TZ and ultimately into the warehouse.

$TD$ degrades as $V$ increases. This holds over all $\lambda$ tested: 65.6, 75.6, and 85.6. As seen in Table 3, 4, and 5, ranging from a low $V$ (4.2) to a medium $V$ (337.5) to a high $V$ (937.5), we observe a significant increase in travel with a constant $\lambda$.  


Table 3: $TD$ for the following parameter values: $\lambda = 75.6$, $SL = 378$ (smaller warehouse), and varying $V$.

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Ratio of Lowest $V$</th>
<th>$TD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.6</td>
<td>80.6</td>
<td>75.6</td>
<td>1.0</td>
<td>983,414</td>
</tr>
<tr>
<td>60.6</td>
<td>90.6</td>
<td>75.6</td>
<td>9.0</td>
<td>1,196,707</td>
</tr>
<tr>
<td>50.6</td>
<td>100.6</td>
<td>75.6</td>
<td>25.0</td>
<td>1,465,819</td>
</tr>
<tr>
<td>40.6</td>
<td>110.6</td>
<td>75.6</td>
<td>49.0</td>
<td>1,754,876</td>
</tr>
<tr>
<td>30.6</td>
<td>120.6</td>
<td>75.6</td>
<td>80.9</td>
<td>2,075,272</td>
</tr>
<tr>
<td>20.6</td>
<td>130.6</td>
<td>75.6</td>
<td>120.9</td>
<td>2,405,290</td>
</tr>
<tr>
<td>10.6</td>
<td>140.6</td>
<td>75.6</td>
<td>168.9</td>
<td>2,743,241</td>
</tr>
<tr>
<td>0.6</td>
<td>150.6</td>
<td>75.6</td>
<td>224.8</td>
<td>3,086,234</td>
</tr>
</tbody>
</table>

Table 4: $TD$ for the following parameter values: $\lambda = 65.6$, $SL = 378$ (smaller warehouse), and varying $V$.

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Ratio of Lowest $V$</th>
<th>$TD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.6</td>
<td>70.6</td>
<td>65.6</td>
<td>1.0</td>
<td>792,256</td>
</tr>
<tr>
<td>50.6</td>
<td>80.6</td>
<td>65.6</td>
<td>9.0</td>
<td>903,464</td>
</tr>
<tr>
<td>40.6</td>
<td>90.6</td>
<td>65.6</td>
<td>25.0</td>
<td>1,122,936</td>
</tr>
<tr>
<td>30.6</td>
<td>100.6</td>
<td>65.6</td>
<td>49.0</td>
<td>1,399,466</td>
</tr>
<tr>
<td>20.6</td>
<td>110.6</td>
<td>65.6</td>
<td>80.9</td>
<td>1,697,563</td>
</tr>
<tr>
<td>10.6</td>
<td>120.6</td>
<td>65.6</td>
<td>120.9</td>
<td>2,019,050</td>
</tr>
<tr>
<td>0.6</td>
<td>130.6</td>
<td>65.6</td>
<td>168.9</td>
<td>2,349,439</td>
</tr>
</tbody>
</table>
Table 5: $TD$ for the following parameter values: $\lambda = 85.6$, $SL = 378$ (smaller warehouse), and varying $V$.

<table>
<thead>
<tr>
<th>Smaller Warehouse</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Ratio of Lowest V</th>
<th>$TD$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80.6</td>
<td>90.6</td>
<td>85.6</td>
<td>1.0</td>
<td>1,306,310</td>
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<td>70.6</td>
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<td>9.0</td>
<td>1,543,629</td>
</tr>
<tr>
<td></td>
<td>60.6</td>
<td>110.6</td>
<td>85.6</td>
<td>25.0</td>
<td>1,824,836</td>
</tr>
<tr>
<td></td>
<td>50.6</td>
<td>120.6</td>
<td>85.6</td>
<td>49.0</td>
<td>2,135,530</td>
</tr>
<tr>
<td></td>
<td>40.6</td>
<td>130.6</td>
<td>85.6</td>
<td>80.9</td>
<td>2,458,977</td>
</tr>
<tr>
<td></td>
<td>30.6</td>
<td>140.6</td>
<td>85.6</td>
<td>120.9</td>
<td>2,796,767</td>
</tr>
<tr>
<td></td>
<td>20.6</td>
<td>150.6</td>
<td>85.6</td>
<td>168.9</td>
<td>3,133,369</td>
</tr>
<tr>
<td></td>
<td>10.6</td>
<td>160.6</td>
<td>85.6</td>
<td>224.8</td>
<td>3,494,621</td>
</tr>
</tbody>
</table>

This increased $TD$ can be immediately attributed to the greater chance of having a large number of incoming unit-containers each day each day. Seeing that $V$ is higher, the minimum is decreased and maximum is increased for $\lambda$. Whenever there is a large number of incoming unit-containers (whenever the actual incoming is close to the maximum), there is a chance there will be more incoming unit-containers than open storage locations. When this occurs there is an added travel penalty associated for unit-containers that do not make it into the warehouse. Because of the increased chance of having more unit-containers than open storage locations, the $TD$ will increase despite the average number of incoming unit-containers staying constant. Observing this, as $V$ continues to increase, it is likely that $TD$ will continue to increase due to this penalty.

This observation can attributed to when there are more incoming unit-containers than open storage locations, those “extra” unit-containers are never actually stored within the warehouse. We assume the “extra” PI-containers are moved to an ambient space and never handled again. Because of this extra travel and inconvenience, the travel penalty is incurred.

$PTZ$ actually improves as $V$ increases because the percentage that makes it into its TZ increases, holding for all three values of $\lambda$. Table 6, 7, and 8 show from a low $V$ to a medium $V$ to a high $V$, we observe about a 50% decrease in this response value, holding $\lambda$ constant.
Table 6: $P_{TZ}$ and $P_{WH}$ for the following parameter values: $\lambda = 75.6$, $SL = 378$ (smaller warehouse), and varying $V$.

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Ratio of Lowest V</th>
<th>% Not in TZ</th>
<th>% Not in WH</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.6</td>
<td>80.6</td>
<td>75.6</td>
<td>1.0</td>
<td>8.8</td>
<td>0.0</td>
</tr>
<tr>
<td>60.6</td>
<td>90.6</td>
<td>75.6</td>
<td>9.0</td>
<td>8.3</td>
<td>3.8</td>
</tr>
<tr>
<td>50.6</td>
<td>100.6</td>
<td>75.6</td>
<td>25.0</td>
<td>7.0</td>
<td>10.9</td>
</tr>
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<td>80.9</td>
<td>5.7</td>
<td>25.7</td>
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<td>75.6</td>
<td>120.9</td>
<td>5.0</td>
<td>33.2</td>
</tr>
<tr>
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<td>4.8</td>
<td>41.0</td>
</tr>
<tr>
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<td>75.6</td>
<td>224.8</td>
<td>4.5</td>
<td>48.8</td>
</tr>
</tbody>
</table>

Table 7: $P_{TZ}$ and $P_{WH}$ for the following parameter values: $\lambda = 65.6$, $SL = 378$ (smaller warehouse), and varying $V$.

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Ratio of Lowest V</th>
<th>% Not in TZ</th>
<th>% Not in WH</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.6</td>
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</tr>
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<td>25.0</td>
<td>8.5</td>
<td>7.8</td>
</tr>
<tr>
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<td>49.0</td>
<td>7.6</td>
<td>8.9</td>
</tr>
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<td>65.6</td>
<td>168.9</td>
<td>5.2</td>
<td>31.9</td>
</tr>
</tbody>
</table>
Table 8: $P_{TZ}$ and $P_{WH}$ for the following parameter values: $\lambda = 85.6$, $SL = 378$ (smaller warehouse), and varying $V$.

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Maximum</th>
<th>Mode</th>
<th>Ratio of Lowest $V$</th>
<th>% Not in TZ</th>
<th>% Not in WH</th>
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<td>7.0</td>
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<td>9.0</td>
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<td>25.0</td>
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</tr>
<tr>
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<td>27.1</td>
</tr>
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<td>80.9</td>
<td>5.0</td>
<td>34.5</td>
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<td>30.6</td>
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<td>120.9</td>
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<td>20.6</td>
<td>150.6</td>
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<td>168.9</td>
<td>4.2</td>
<td>49.9</td>
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<tr>
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<td>160.6</td>
<td>85.6</td>
<td>224.8</td>
<td>3.6</td>
<td>58.1</td>
</tr>
</tbody>
</table>

As $V$ increases, so does the chance there will be more incoming unit-containers than open storage locations as stated earlier. This observation can be directly seen in $P_{WH}$. As $V$ increases, this percentage increases as well. Considering a $\lambda = 75.6$, with a ranging variation, $P_{WH}$ increases from 3% up to about 49% on average for a day.

Because on days where there are more incoming unit-containers than open storage locations, there will be less unit-containers actually stored. The warehouse will then have a higher percentage of unit-containers with a lower DOS. Knowing this, we acknowledge these unit-containers will be shipped sooner than if all of the previous unit-containers were stored in the warehouse. Because the containers are shipped quicker, there ends up being a higher number of open storage locations within the warehouse from day-to-day, especially in the days after a day with excess incoming unit-containers. Considering there are more open spots because the extra unit-containers do not make it into the warehouse, consequently there is a greater chance of future unit-containers will make it into the warehouse and ultimately into its TZ.

4. For very high/low levels of $V$, how does $TD$ respond to an increase in $\lambda$?

As $V$ increases, the percent increase in $TD$ decreases whether it is a high or low $\lambda$. Essentially, there is a difference in the way the $TD$ responds to changes in the $\lambda$ when there is a high or low $V$. Figure 9 expresses these changes.
For the highest level of \( V \) tested, there was a 17\% increase in \( TD \) for a 15\% increase in \( \lambda \) (from 65.6 to 75.6). This is compared to a 24\% increase in travel for the lowest level of \( V \). This same effect is observed for a slightly lower increase in \( \lambda \). For a 13\% increase in \( \lambda \) (from 75.6 to 85.6) there is a 14\% increase in \( TD \) for a high \( V \) when compared to a 32\% increase in travel for a low \( V \).

As \( V \) increases, the increase in \( TD \) becomes decreasingly evident. In other words, the \( TD \) increases by a smaller and smaller percentage as \( V \) increases. The effect the increasing \( \lambda \) has on the \( TD \) is lessened as \( V \) increases. Considering this, if there is a high \( V \) then a change in \( \lambda \) will have a smaller and smaller impact on \( TD \).

5. Given a DOS policy in the PI, do larger warehouses produce any advantage?

Is there a gained level of efficiency with a larger warehouse? We compared two different size warehouses: 30,240 ft\(^2\) (smaller) and 145,920 ft\(^2\) (larger). The ratio of the larger warehouse to the smaller warehouse is 4.83, so essentially the larger warehouse is a little larger than four and half times the smaller warehouse. When considering this ratio in square footage, we would like to see if \( TD \) for each warehouse size reflects this size difference as well. To appropriately
compare the two different warehouses, we consider the average one-way travel per put-away of a unit-container rather than $TD$.

We want to compare the two different size warehouses with the same ratio of $SL$ to $\lambda$. This ratio is equal to 5 because the average DOS stays consistent over both size warehouses. One-way travel per put-away of a unit-container example:

$$\frac{TD}{(\lambda \times \text{run length})} = \frac{8,702,367.76}{316.16(210)} = 131.1$$

For the larger warehouse and $\lambda = 316.16$, the travel distance per unit-container ranges from 131.1 feet to 133.0 feet to 134.8 feet for low, medium, and high levels of $V$, respectively. This 131.1 feet represents the average one-way travel required to put away a unit container with the following parameters: $V = 4.17$, $\lambda = 316.16$, and $SL = 1,824$.

For the smaller warehouse ($SL = 378$, $\lambda = 65.6$), the travel distance per unit-container ranges from 57.5 feet to 68.7 feet to 79.9 feet for low, medium, and high levels of $V$, respectively. The 57.5 feet in travel represents the average one-way travel required to put away a unit container with the following parameters: $V = 4.17$, $\lambda = 65.6$, and $SL = 378$.

To better view the effects of scale in $TD$ between the two warehouses, it is better to keep all other parameters consistent except for the size of the warehouse. When keeping these other parameters consistent, we considered the ratio between the travel distances per unit-container for the larger warehouse to the smaller warehouse. We consider how this might compare to the ratio of the overall square footage from one warehouse to the other (4.83). These ratios range from 2.28 to 1.94 to 1.69 for low, medium, and high levels of $V$, respectively. The full analysis for these travel distances and ratios can be seen in Figures 10 and 11.
Figure 10. Average Travel Distance per Unit-Container for Medium $\lambda$.

Figure 11. Ratio of the Average Travel Distance per Unit-Container (Large to Small)
It is obvious at first glance these values change along with $V$. This can be attributed to the fact that the change in $V$ has little comparative effect on the larger warehouse. The travel distance per unit-container for the larger warehouse slightly changes with changes in $V$, but the smaller warehouse produces a more significant change. Basically, the percent increase in $TD$ for an increasing $V$ is not equal for the smaller and larger warehouse. For the smaller warehouse, an increase in $V$ would significantly increase the travel distance per unit-container. On the other hand, for the larger warehouse, we only see a slight increase in the travel distance per unit-container in response to an increase in $V$.

The $P_{WH}$ increases due to the increase in warehouse size as seen in Figure 5. As more storage locations are added to the warehouse, $\lambda$ also increases; so more unit-containers will arrive each day as well. With this increase in $\lambda$ comes a greater variation in the number of incoming unit-containers. Because of this greater variation, there will be a greater chance of receiving more unit-containers than open storage locations. Because of the decrease in $P_{WH}$, a less percentage of unit-containers are successfully stored in the warehouse.

In response to an increase in warehouse size, $P_{TZ}$ decreases. Although a less percentage of unit-containers are successfully stored in the warehouse, a greater percentage of unit-containers are successfully stored in their TZ, for the larger warehouse. As previously discussed, the reason for the decrease in $P_{TZ}$ stems from the analysis of our assumption that all “extra” incoming unit-containers are stored in an ambient space and never handled again. The days following days of high volume will yield a lower $P_{TZ}$.

$P_{WH}$ increases as the warehouse size increases. As more storage locations are added to the warehouses, $\lambda$ also increases. Along with these two increases, a less percentage of unit-containers are successfully stored in the warehouse. After days of high activity, there will be more open storage locations within the warehouse because a high number of unit-containers never actually made it into the warehouse during those days of high activity. Because of this increase in open storage locations, there stands a better chance that unit-containers will be stored in its target zone, resulting in a decrease in $P_{TZ}$.

When considering the warehouse size, it is vital for a warehouse operator to identify what is most important in their operation. If the warehouse operator has the ability to divert a high number of unit-containers to another facility on a daily basis, then a larger warehouse is allowable because of the gained efficiency in $P_{TZ}$. If the overall goal of the warehouse operation
is to always successfully store each unit-container received, then it is preferred to have multiple smaller warehouses rather than just one larger warehouse.

6. CONCLUSIONS

We want to be able to use all of this information to aid in the decision-making process associated with the design and implementation of a warehouse and storage policy in the PI. We want to make decisions in light of observations from all the questions we considered. The observations and answers from the results aid in various different decisions.

These results will help to decide different initial aspects of the warehouse such as the size of the warehouse and the size of each of TZ. Given the volume of the throughput, a warehouse operator will be able to use the results from Questions 1 and 5 to see what their system might look given the size of the warehouse they will build. The warehouse operator will be able to compare a “right-sized” warehouse with a slightly larger warehouse given the observations and results. For the warehouse operator, is it worth it for them to take on the costs with building a slightly larger warehouse? For the larger warehouse, we see that the TD is less affected by changes in \( \lambda \) and \( V \). So, although this would have an extra upfront land and building cost, the warehouse operator would be able to better predict associated travel costs even when the system is unpredictable.

A warehouse operator would also be able to use information from Question 1 to help in the decision of sizing each TZ. Results from Question 1 helps to show how the percentage not in the TZ or the warehouse changes as the warehouse gets larger. It would be helpful for a company to know how efficient the TZ would be in comparison to the size of the warehouse.

Even though our results help with initial decisions, our results also aid with ongoing decisions arising as different parameters of the warehouse change with time. The number of incoming unit-containers will probably vary from one week or day to the next. The warehouse operator will need to know how their system will react in response to these changes in \( \lambda \). Through this data, the operator would then be able to make a smart, well thought-out decision.

Along with the change in the average number of incoming unit-containers, the warehouse manager is bound to see a change in the variation in number of incoming containers. The minimum and maximum number of incoming containers will always be changing, so it would be important to consider the how important values will change, such as the efficiency of the TZs
and the overall $TD$. An example being, if the manager knows that if the variation of the number of incoming containers is about to increase, per our results, they will also know their $TD$ will also increase. Given this increase, it might prove to be impossible to complete all put-aways with the given level of workforce. In response to this, the warehouse manager might need to hire more operators.

In short, the questions and results that have come from our research will help in deciding a type of business model for a warehouse manager. A warehouse manager will need help in deciding what type of storage policy would be best for their system.

Whereas the PI is still being shaped, there are still a few more questions to be considered and answered. As of now there has been no research or observations of what might happen if the unit-containers were strategically moved around during the night shifts. For most instances, the manager would want to move the unit-containers that were originally stored in the wrong zone. These unit-containers would then be moved from this wrong zone to their target zone. This could create savings in retrievals the next morning because it could require less travel to retrieve the containers. It could always require more travel in the case that the unit-container was moved to a zone farther away. On other hand, it would most likely require increased travel to put away the incoming unit-containers because the open locations would be farther away. One other question to consider might be would we want to purposefully oversize the facility. If the picking area were slightly oversized, there would be a higher number of open locations for PI-containers to be stored. This would hurt somewhat because the facility would be bigger than it has to be, but it would increase the likelihood that a location is open when a PI-container arrives. To actually implement the oversizing of the facility, one would want to make sure that the cost from having a larger facility is outweighed by the increased efficiency from having more “good” locations open. Among others, the analysis of this idea would be an interesting future research project about a DOS storage policy in the PI.
References